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Electron tunneling in heterostructures under a transverse magnetic field

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Abstract : The transfer matrix formalism is used to study the electron tunneling in semiconductor heterostructures in the presence of a transverse magnetic field. The transmission coefficients for heterostructures where the barriers are arranged in a manner either periodic or quasiperiodic are calculated. In a quasiperiodic heterostructure, the group of resonant peaks is depressed relative to the resonant peaks in a periodic heterostructure. The magnetic field produces a shift of the transmission coefficient to a higher energy value and, when the field increases, the peaks in the group of resonances are depressed progressively and finally disappear in a stronger magnetic field.

Keywords : Transmission, heterostructures, transverse magnetic field

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1. Introduction

Recent advances in submicrometer physics have made possible the fabrication of low-dimensional electronic systems (Roukes *et al* 1989). This has naturally stimulated interest in their physical properties, especially those related to transport phenomena. There have been numerous studies, both experimental and theoretical, devoted to the physics of transport in semiconductor heterostructures under a variety of conditions related with temperature, electric and magnetic fields, dimension, arrangement and many-body interactions (Büttiker 1988, Landauer 1989, Harris *et al* 1989). In particular, electron tunneling through a heterostructure in a transverse magnetic field has been studied extensively (Ando 1981, Xia and Fan 1989, Helm *et al* 1989, Cruz *et al* 1990, Zaslavsky *et al* 1990, Curry *et al* 1990).

Hung and Wu (1992) have considered the $GaAs/Al_xGa_{1-x}As$ heterostructure and obtained the energy levels and electron tunneling in such a heterostructure under an in-plane magnetic field. At the same time, transmission through a one-dimensional (1*D*) quasiperiodic system has attracted considerable attention (Würtz *et al* 1988, Avishai and Berend 1990, 1991). Singh *et al* (1992) have made a comparative study of electron tunneling in periodic and quasiperiodic superlattice systems. Recently, Chen *et al* (1994) studied the electron tunneling in the semiconductor quantum-wire superlattice with randomly distributed layer thicknesses.

In this article we study the transport properties of a semiconductor heterostructure in a transverse magnetic field. We make quantum-mechanical calculation of the electron tunneling in heterostructures under a transverse magnetic field. To calculate the transmission coefficient we solve the Schrödinger equation in one cell and then by the successive multiplications of the transfer matrices we obtain transmission and reflection amplitudes for the whole structure. We calculate the transmission coefficient for a heterostructure where the barriers are arranged in a manner either periodic or quasiperiodic and compare the results for these two cases.

The remaining part of the paper is organized as follows. Section 2 introduces the heterostructure under study and contains the theoretical formalism used in our calculation. In section 3 we give the results with detailed discussions. Section 4 is a summary.

2. Theoretical formalism

Here we consider semiconductor heterostructures in which each building block consists of double layers. We further assume that the first (second) layers are constituted by the same semiconductor material *e.g.* by GaAs ($Al_xGa_{1-x}As$). In the presence of a transverse magnetic field, the Hamiltonian can be written as

$$H = \frac{1}{2m^*}\left[p_x^2 + (p_y + eA)^2 + p_z^2\right] + U(x), \tag{1}$$

where m^* is the electron effective mass and the building blocks are assumed to be arranged along the x direction. For the gauge A associated with the magnetic field, it can be written as $A = (0, 0, 0)$ for $x < 0$, $A = (0, Bx, 0)$ for $0 < x < L$, and $A = (0, BL, 0)$ for $x > L$, where L is the size of the system in the x direction. Substituting the wave function

$$\Psi(x, y, z) = e^{i(k_y y + k_z z)}\varphi(x) \tag{2}$$

into the Schrödinger equation $H\Psi = E\Psi$, one obtains the eigenvalue equation

$$-\frac{\hbar^2}{2m^*}\left[\frac{d^2}{dx^2} - \left(k_y + \frac{eBx}{\hbar}\right)^2\right]\varphi(x) + U(x)\varphi(x) = \left[\left(E - \frac{\hbar^2 k_z^2}{2m^*}\right)\varphi(x)\right]. \tag{3}$$

In what follows, we use the Kronig-Penney model to characterize the potential $U(x)$, *i.e.* the potential is assigned as constants 0 and V within the first and second layers (corresponding to the well and barrier) of each building block, respectively.

We divide the i-th well (barrier) into $M(N)$ slabs and treat the gauge A within every single slab as a constant vector (Taylor 1977). For explicitness, in the j-th slab of the i-th well, $(x_i + ja_i/M, x_i + (j+1)a_i/M)$, $j = 0, 1, 2, \ldots, M-1$, where a_i is the width of the i-th well, the term $\left[k_y + (eB/\hbar)x\right]^2$ in eq. (3) is approximated by $\left[k_y + (eB/\hbar) \times (x_i + ja_i/M)\right]^2$ and within this slab eq. (3) then becomes

$$-\frac{\hbar^2}{2m^*}\frac{d^2\varphi(x)}{dx^2} + \frac{\hbar^2}{2m^*}\left[k_y + \frac{eB}{\hbar}\left(x_i + j\frac{a_i}{M}\right)\right]^2 \varphi(x)$$
$$= \left(E - \frac{\hbar^2 k_z^2}{2m^*}\right)\varphi(x), \tag{4}$$

which has the plane-wave solution

$$\begin{aligned}\varphi_{i,j}(x) &= c_{i,j}\, e^{ik_{i,j}(x - (x_i + ja_i/M))} + d_{i,j}\, e^{-ik_{i,j}(x - (x_i + ja_i/M))},\\ x &\in \left(x_i + ja_i/M,\ x_i + (j+1)a_i/M\right).\end{aligned} \tag{5}$$

where $k_{i,j}$ is given by

$$k_{i,j} = \left\{\frac{2m^* E}{\hbar^2} - k_z^2 - \left[k_y + \frac{eB}{\hbar}\left(x_i + j\frac{a_i}{M}\right)\right]^2\right\}^{1/2}. \tag{6}$$

Within the j-th slab of the i-th barrier, $\left(x_i + a_i + jb_i/N, x_i + a_i + (j+1)b_i/N\right)$, $j = 0, 1, 2, \ldots, N_i - 1$, where b_i is the width of the i-th barrier, eq. (3) is approximated by

$$-\frac{\hbar^2}{2m^*}\frac{d^2\varphi(x)}{dx^2} + \frac{\hbar^2}{2m^*}\left[k_y + \frac{eB}{\hbar}\left(x_i + a_i + j\frac{b_i}{N}\right)\right]^2 \varphi(x)$$
$$= \left(E - V - \frac{\hbar^2 k_z^2}{2m^*}\right)\varphi(x) \tag{7}$$

with the plane-wave solution

$$\begin{aligned}\Phi_{i,j}(x) &= C_{i,j}\, e^{iK_{i,j}(x - (x_i + a_i + jb_i/N))} + D_{i,j}\, e^{-iK_{i,j}(x - (x_i + a_i + jb_i/N))},\\ x &\in \left(x_i + a_i + jb_i/N,\ x_i + a_i + (j+1)b_i/N\right),\end{aligned} \tag{8}$$

where
$$K_{i,j} = \left\{\frac{2m^*(E-V)}{\hbar^2} - k_z^2 - \left[k_y + \frac{eB}{\hbar}\left(x_i + a_i + j\frac{b_i}{N}\right)\right]^2\right\}^{1/2}. \tag{9}$$

After the above approximation, the gauge A within the heterostructure is replaced by a *stair-step* vector potential. When the number of the slabs in each well (barrier) is sufficiently large, eqs. (4) and (7) will accurately characterize the behaviour of the electron in the heterostructure.

From the wave-function-matching conditions at the boundaries of the slabs along the x direction, a set of coupled equations linking the amplitudes $\{c_{i,j}, d_{i,j}\}$ and $\{C_{i,j}, D_{i,j}\}$ can be derived :

$$\begin{pmatrix} c_{i,j+1} \\ \\ d_{i,j+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\left(1+\frac{k_{i,j}}{k_{i,j+1}}\right)e^{ik_{i,j}a_i/M} & \frac{1}{2}\left(1-\frac{k_{i,j}}{k_{i,j+1}}\right)e^{-ik_{i,j}a_i/M} \\ \frac{1}{2}\left(1-\frac{k_{i,j}}{k_{i,j+1}}\right)e^{ik_{i,j}a_i/M} & \frac{1}{2}\left(1+\frac{k_{i,j}}{k_{i,j+1}}\right)e^{-ik_{i,j}a_i/M} \end{pmatrix} \begin{pmatrix} c_{i,j} \\ \\ d_{i,j} \end{pmatrix}, \tag{10}$$

$$j = 0, 1, 2, \ldots, M-2,$$

$$\begin{pmatrix} C_{i,0} \\ \\ D_{i,0} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\left(1+\frac{k_{i,M-1}}{K_{i,0}}\right)e^{ik_{i,M-1}a_i/M} & \frac{1}{2}\left(1-\frac{k_{i,M-1}}{K_{i,0}}\right)e^{-ik_{i,M-1}a_i/M} \\ \frac{1}{2}\left(1-\frac{k_{i,M-1}}{K_{i,0}}\right)e^{ik_{i,M-1}a_i/M} & \frac{1}{2}\left(1+\frac{k_{i,M-1}}{K_{i,0}}\right)e^{-ik_{i,M-1}a_i/M} \end{pmatrix} \begin{pmatrix} c_{i,M-1} \\ \\ d_{i,M-1} \end{pmatrix}, \tag{11}$$

$$\begin{pmatrix} C_{i,j+1} \\ \\ D_{i,j+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\left(1+\frac{K_{i,j}}{K_{i,j+1}}\right)e^{iK_{i,j}b_i/N} & \frac{1}{2}\left(1-\frac{K_{i,j}}{K_{i,j+1}}\right)e^{-iK_{i,j}b_i/N} \\ \frac{1}{2}\left(1-\frac{K_{i,j}}{K_{i,j+1}}\right)e^{iK_{i,j}b_i/N} & \frac{1}{2}\left(1+\frac{K_{i,j}}{K_{i,j+1}}\right)e^{-iK_{i,j}b_i/N} \end{pmatrix} \begin{pmatrix} C_{i,j} \\ \\ D_{i,j} \end{pmatrix}, \tag{12}$$

$$j = 0, 1, 2, \ldots, N-2$$

$$\begin{pmatrix} c_{i+1,0} \\ \\ d_{i+1,0} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\left(1+\frac{K_{i,N-1}}{k_{i+1,0}}\right)e^{iK_{i,N-1}b_i/N} & \frac{1}{2}\left(1-\frac{K_{i,N-1}}{k_{i+1,0}}\right)e^{-iK_{i,N-1}b_i/N} \\ \frac{1}{2}\left(1-\frac{K_{i,N-1}}{k_{i+1,0}}\right)e^{iK_{i,N-1}b_i/N} & \frac{1}{2}\left(1+\frac{K_{i,N-1}}{k_{i+1,0}}\right)e^{-iK_{i,N-1}b_i/N} \end{pmatrix} \begin{pmatrix} C_{i,N-1} \\ \\ D_{i,N-1} \end{pmatrix}. \tag{13}$$

By successive multiplications of the transfer matrices given in eqs. (10–13), we are able to to obtain the transfer matrix ($M = (m_{ij})$ linking the amplitudes of the wave functions at the left and right ends of the structure, and finally calculate the transmission coefficient by the following equation :

$$\begin{pmatrix} t \\ 0 \end{pmatrix} = M\begin{pmatrix} 1 \\ r \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}\begin{pmatrix} 1 \\ r \end{pmatrix}, \tag{14}$$

where $r(t)$ is the amplitude of the reflected (transmitted) plane wave at the left (right) end of the structure. Here, the amplitude of the plane wave incident to the structure is chosen to be 1. From eq. (14) it follows that the reflection coefficient is $R = |r|^2 \quad \frac{m_{21}}{n_{22}}$ and the transmission coefficient is thus given by $T = 1 - R = S - \frac{n_{21}}{m_{22}}$ according to the law of probability conservation.

3. Numerical calculations

We first consider periodic heterostructures in which two wells with widths a_L and a_R are at their left and right ends, and the widths of barriers and other wells take values a and b, respectively. In our numerical calculations, we use dimensionless quantities, *i.e.*, the energy and magnetic field are in units of $\hbar^2\pi^2/2m^*b^2$ and $\hbar/eb^2$ respectively. When the transverse magnetic field is applied, we divide every well (barrier) into $M(N) = 100$ slabs in the numerical calculations. In Figure 1 we present the transmission coefficcient for the double-barrier case in which $a_L = a_R = 0.25$, $a = 0.5$, $b = 1$, and $V = 1$; the magnetic field is chosen to be $B = 0$, 0.02, 0.06 and 0.1, corresponding to the solid, dashed, dotted and dash-dotted curves, respectively. Also, k_y and k_z are both taken to be zero. From Figure 1 one sees that

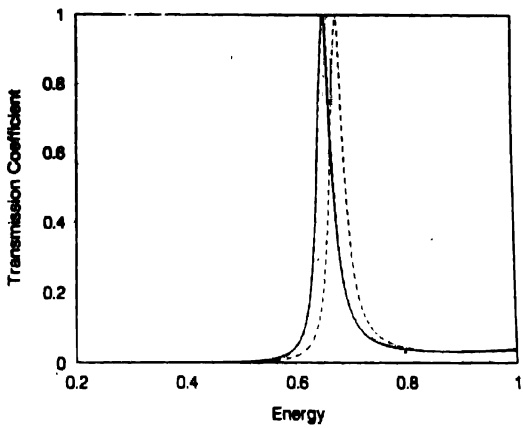

Figure 1. Transmission coefficient for a double-barrier structure, where $a_L = a_R = 0.25$, $a = 0.5$, $b = 1$ and $V = 1$ and $B = 0$ (solid curve), 0.02 (dashed curve), 0.06 (dotted curve) and 0.1 (dash-dotted curve). Also k_y and k_z are both taken to be zero

only one resonant peak occurs in the considered energy range and the peak shifts rightward as the magnetic field increases. This observation matches the results obtained by Hung and Wu (1992) for the double-barrier heterostructure. In Figures 2(a)–2(d) the transmission coefficient is calculated for a periodic heterostructure with five barriers, in which the parameters are chosen to be the same as in Figure 1 and $B = 0$, 0.04, 0.07 and 0.1, respectively. It can be seen that with the increase of the magnetic field, the resonant-peak group shows an overall rightward shift and the resonant peaks are depressed. Particularly, when the magnetic field is strong enough, a given peak can even be completely depressed. We have also calculated the transmission coefficient for a periodic heterostructure with 13 barriers (see Figure 3), where the parameters are the same as in Figure 1 and $B = 0$ and 0.02. Also, it can be seen that there exists apparent depression of the resonant peaks as induced by the applied maganetic field.

Finally, we study a heterostructure with the barriers arranged in a quasiperiodic manner. The parameters of the structure are chosen to be $a_L = a_R = 0.25$, $a = 0.5$ and $V = 1$, as in the case of periodic heterostructures, but the width of each barrier takes the value of either $b_A = 1.2$ or $b_B = 0.8$.

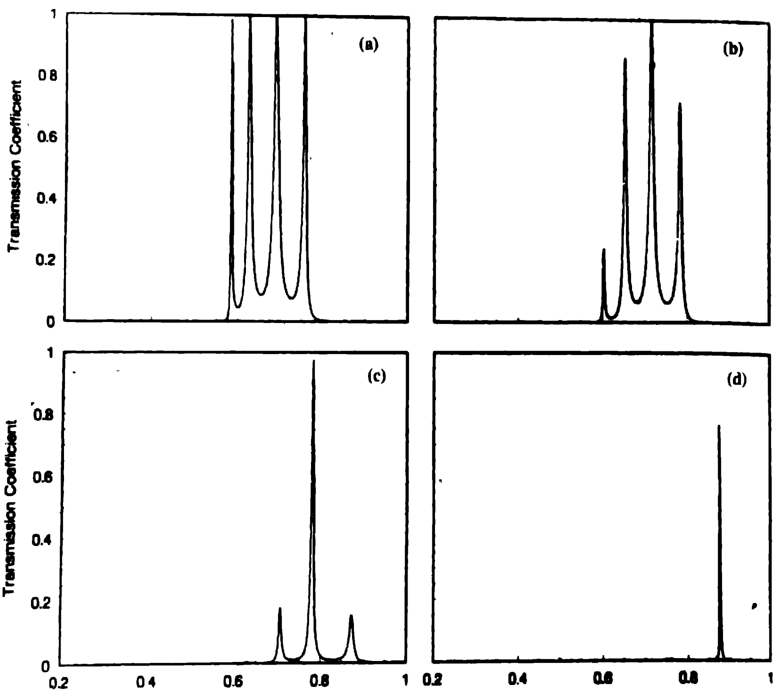

Figure 2. Transmission coefficient for a periodic structure with five barriers, where the parameters are the same as in Figure 1 and B = (a) 0, (b) 0.04, (c) 0.07 and (d) 0.1

The barriers are arranged according to the construction rule for the Fibonacci sequence (Kohmoto *et al* 1987) : $S_{l+1} = \{S_l, S_{l-1}\}$ with $l \geq 1$ and the initial conditions $S_0 = \{B\}$ and $S_1 = \{A\}$. For this construction rule, the number of letters A and B in S_l obeys the recursion relation $F_{l+1} = F_l + F_{l-1}$ with $F_0 = F_1 = 1$. Figures 4(a) and 4(b) show the transmission coefficient for a quasiperiodic heterostructure with $F_6 = 13$ barriers, where the magnetic field is chosen to be $B = 0$ and 0.02, respectively. In the absence of the magnetic field, the group of resonances is not as high as in the periodic case [comparing Figure 4(a) with Figure 3(a)]. This overall depression of the resonant peaks is due to the quasiperiodic order existing in the heterostructure. When the magnetic field is applied, the transmission coefficient shifts rightward and the field-induced depression of the resonant peaks also occurs.

4. Summary

In summary, we have studied the effect of a transverse magnetic field on the electronic transmission in semiconductor heterostructures. The transfer matrix approach is employed.

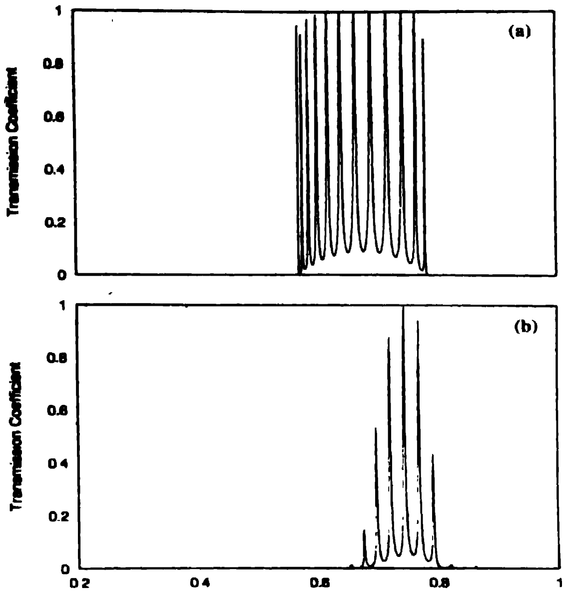

Figure 3. Transmission coefficient for a periodic structure with 13 barriers, where the parameters are the same as in Figure 1 and B = (a) 0 and (b) 0.02.

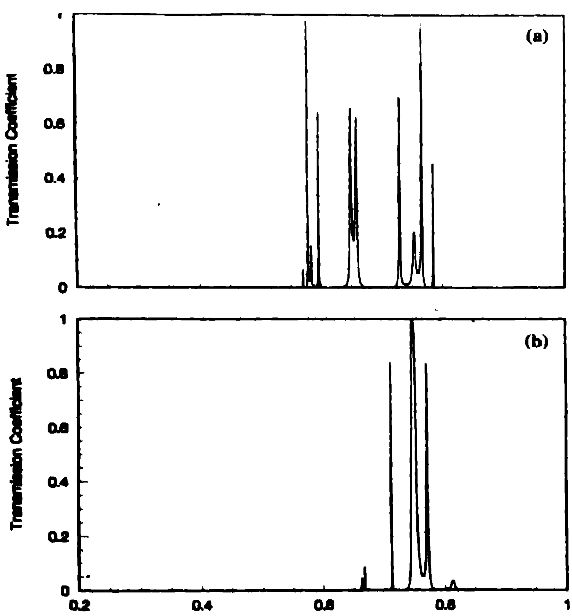

Figure 4. Transmission coefficient for a quasiperiodic structure with 13 barriers, where the parameters are the same as in Figure 1 but the width of each barrier takes either b_A = 1.2 or b_B = 0.8, and B = (a) 0 and (b) 0.02.

The heterostructures under study have the barriers arranged in a manner either periodic or quasiperiodic. We have compared the results for quasiperiodic system with those of the periodic system both in the presence or absence of a magnetic field. We have employed a plane-wave transfer matrix formalism in this work, while the parabolic-cylinder-function transfer matrix was used by Hung and Wu (1992). Our approach is more efficient and less cumbersome, and the results become accurate when each of the wells and barriers is divided into a large number of slabs. For a periodic double-barrier heterostructure there is a single resonant peak [in the energy range considered], while there is a number of peaks in a group for a heterostructure with more barriers. The presence of the magnetic field results in an overall rightward shift of the resonant peak group. With increase of the magnetic field, the resonant peaks are depressed progressively and are totally depressed in a strong magnetic field. For the quasiperiodic heterostructure, the group of resonances is depressed relative to the resonant peaks in the periodic case. This is due to the quasiperiodic order existing in the heterostructure. With the application of the magnetic field, the transmission coefficient shifts to a higher energy value and the resonant peaks are depressed.

Acknowledgments

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